"Many paths" Quantum Physics and Classical Electromagnetism

Many teachers and students of *Advancing Physics* have raised interesting queries about the relationship between the "many paths" story about photons, and the classical story of electromagnetic fields and waves. For example:

> "What can we say about how the polarisation of electromagnetic waves fits into the 'many paths' story?"
>
> "How does exchange of photons give rise to the electrical force between charges? How does the dependence of the force on the charge of the particles come about?"

The notes below are an attempt to say briefly what the answers to such questions look like, and to explain why they are not always easy to answer.

Classical and quantum

It's important to remember that photons don't carry classical properties, like polarisation, 'in miniature' as it were. A well-defined electromagnetic field only exists when there are very large numbers of photons. The phase of the wave has no meaning unless there are many photons. So we must look for the links in this limit.

Fictitious spin-zero photons

The photons in Feynman's QED book, and in Advancing Physics, are fictions. Their spin is not taken into account. So they are in effect spin-zero, mass-zero particles. No such particles are known. But it keeps the story simple, which was the point. This is why polarisation, which is an effect of the spin of the photons (spin 1), does not appear in the story.

Polarisation and relativity

Making sure that a quantum field theory is relativistically invariant has important consequences.
One is that, because the four dimensions (time plus three space dimensions) must be treated on a par, a photon has **four** 'polarisation' directions. That is, its spin can be projected along any of the four dimensions.
However, for propagation over long distances (i.e. like electromagnetic waves) the energies of the spin components along the direction of propagation and along the time axis cancel exactly, leaving only two transverse polarisations. When there are many photons, the interaction behaves as if it comes from a transversely polarised wave of electromagnetic field.

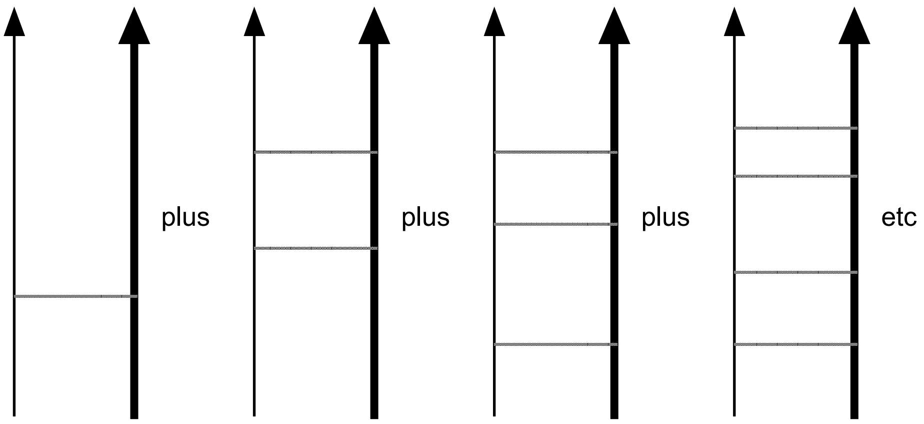
However, it doesn't work like this for the virtual photons mediating the electrical force between pairs of charges at rest, for example. Nor, of course, is the static electric field between the charges polarised transversely in the classical picture. The virtual photon story says that here it is only the 'time-like' polarisation that matters. (As you should expect for a static field, 'moving' only in time.) You may recall a parallel distinction in classical electromagnetic theory between 'near field' (e.g. responsible for electromagnetic induction) and 'far field', or propagating electromagnetic waves.

All this more or less re-states what Feynman wrote in QED, pages 120 to 121.

Electric field between charges

How does the electric field arise from exchange of virtual photons? Feynman sketches the basic ideas on pages 122-123.

Charged particles at rest or travelling slowly can be sketched in Feynman diagrams as going vertically up the page (travelling in time). If one charged particle is massive (e.g. a nucleus) and both travel slowly (non-relativistically), the only Feynman diagrams that contribute a substantial amplitude are those where only a virtual photon is exchanged. Also, the amplitude for exchange of one virtual photon does not depend on the exchange of any other. These diagrams can contain any number of virtual photon exchanges, so look like a series of ladders (and are called "ladder diagrams").



The total amplitude has to be found from the sum (taking account of phase) over all these diagrams. The sum must be taken over all space-time coordinates and all possible energies and momenta, and averaged over the motion of the particles. That is, "try all paths". Notice that the interaction is NOT all contained in one simple diagram, with one photon exchanged between a pair of charged particles.

(Diagrams in which the photon creates an electron positron pair midway, which then annihilate again, contribute a negligible amplitude because of the large energy involved.)

When the sum is done, the interaction turns out to behave as if the massive particle has created an electric field (electric potential proportional to its charge and inversely proportional to distance), with which the other particle interacts. This is called the "external field approximation".

In this calculation, the only term of the electromagnetic potential that survives is the static electric potential. This is the 'time' part of the electromagnetic four-vector. It is in this sense that Feynman is right to say that the 'time-directed' polarisation is the one that matters here.

The **signs** of the two charges enter the expressions for the photon exchange amplitudes, which involve their product. Thus the amplitude is reversed in sign (180 degree phase change) if one charge changes sign. In the end, this makes the energy of the interaction depend on the signs of the charges, whence the rule "like charges repel; unlike charges attract".

If asked, "What's special about positrons that makes them attract electrons instead of repelling them?", the only truthful answer takes this rather opaque form. It happens because opposite signs of charge give 'opposite' phases to the quantum amplitudes for interaction with a photon.

Another way to put it is that "electric charge" simply *is* (in units with $h/2\pi = c = 1$) the measure of the strength of the photon-electron interaction. The sign of the charge gives the phase.

These notes draw on the discussion in Weinberg The Quantum Theory of Fields Vol I, section 13.6. (*really* heavy duty stuff, not for the fainthearted). Feynman points out in QED p 122 that a similar result holds for large numbers of electrons travelling slowly side by side.

The essentials of the connection:

As discussed in Advancing Physics (AS page 174) a very large number of photons propagating from a source to a receiver behave **as if** there is a classical transverse electromagnetic wave with specific frequency and phase.

Now, instead, we can say that a large number of virtual photons exchanged between two charged particles behave **as if** one particle creates a classical static electric field which acts on the other.

From Quantum Physics to Electromagnetism

To conclude, an even tougher section which tries to look at the issues as a whole, from the most fundamental view possible.

The whole of electromagnetism, that is, Maxwell's equations, can be derived from quantum field theory. You start by assuming that there exists a quantised field "carried" by bosons of spin 1 and mass zero. That's all. Then you assume just two principles:

1 Special relativity: that the equations of any theory must be invariant under Lorentz transformations, keeping the speed of light constant. In particular, the dimensions of space and time must be treated exactly on a par. Lorentz invariance then requires there to be a four-vector potential (scalar potential ϕ, and three components of vector potential $\mathbf{A}$), to describe the photons. So the theory starts to look like Maxwell's theory.

2 Gauge invariance. This is trickier to explain. It is related to the simpler fact that the electric potential has no natural zero. It arises because Lorentz invariance does not define the four-potential uniquely, in the argument above. There are four components and only three field equations. So knowing the four-potential at one place and time does not let you determine the values at other places and times. This is because you can add to it the derivative of any arbitrary function of the four co-ordinates, without spoiling the Lorentz invariance or changing the value of the four-potential at one time and place.

As a result, the way forward in constructing a theory is to require that the action be invariant under adding the derivative of an arbitrary function to the four-potential. Much then follows. The gauge symmetry (invariance) requires that there be a conserved charge and current, from Noether's theorem (every symmetry implies a conservation). And the equations for the four-potential become exactly Maxwell's equations, with the conserved charge and current playing the role of electric charge and current.

Thus, from this point of view, the Maxwellian behaviour of spin-1, mass zero bosons comes directly out of relativity and gauge invariance. The existence and conservation of charge is a consequence of gauge invariance.

Mathematical talk and reality talk

One difficulty in responding to perfectly sensible-seeming questions about quantum physics is the fact that much of our talk about quantum physics is mathematical talk masquerading as reality talk. For example:

Mathematical talk: Draw all possible Feynman diagrams (up to a certain complexity) for a given process. To find the probability amplitudes for transitions between 'in'- and 'out'-states, construct (using Feynman rules) for each diagram the integral expression for the amplitude and phase, for any space-time and four-momentum coordinates. Integrate over space-time and momentum coordinates, and sum over all Feynman diagrams.

Reality talk: The particles go anywhere they like, at any speed (energy-momentum) they like. Each path has an associated spinning arrow. Add up the arrows for all the possibilities, to get the final arrow for the whole process.

Something similar goes on in talking about polarisation. Mathematically, the photon is described by a four-vector (the vector potential), and has four components. Relativity requires these to be treated on a par. So the four components get talked about as 'four polarisations'.

This cross-talk gets as bad as it gets when trying to say why only the two transverse polarisations emerge. As it happens, the 'reason' is very much a mathematical artefact. To do the mathematics of gauge invariance you have to "choose a gauge". The choice of gauge is arbitrary. One choice is "Coulomb gauge" (div $\mathbf{A} = 0$). In this gauge, the polarizations in the 'time' direction and in the direction of propagation are both identically zero. The "most fundamental" choice is Lorentz gauge, which keeps everything covariant, unlike the others. Now there are four polarisations, and it is a matter of detailed calculation to see how they work out. For long distance propagation (i.e. e-m waves) the polarisations in the time direction and in the direction of propagation have zero total energy (they cancel identically). The result is the same, but what seems to be 'going on under the bonnet' does not.

Pity, but there it is.