A parabola from the principle of least action

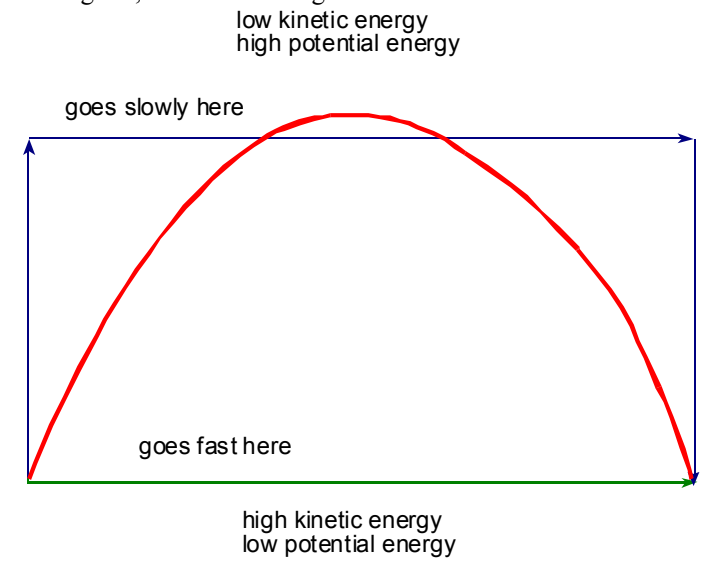
The reading 7R10W ‘Th e Principle of least action’ discusses in outline how a parabola can be thought of, arising from thinking about the sum of (kinetic – potential energy) over several possible paths. Here you get a chance to manipulate an intelligible model, to see how this comes about.

What you need

- A computer running Modellus 2.0 or higher
- the least action model

You make a parabola

Open the model.
When you get to see the Animation window you see two end points which are fixed. You can imagine allowing the ball to try lots of different paths, three of which are shown in this diagram, from the reading:



Action = sum of L×dt.
L = kinetic energy - potential energy

This path gets there in a very short time, but L is too big, because the kinetic energy is large and the potential energy small.
The action is large

This path has a small value of L because much of it is where the potential energy is large and the kinetic energy is small. But it takes a very long time, because it crawls slowly across the top.
The action is large

This path turns out to be ‘just right’ having the least possible action. It balances keeping low and going fast against going high but slowly.
The action is the least possible.

Start the model. Drag the ends of each segment to try out different possible paths. Keep an eye on the level indicator at the bottom of the screen – this shows the total action. You aim to minimse this – and then see what shape you get!
To make the model manageable the horizontal velocity is fixed, allowing you to vary how the height varies with time only.

Four possible starting paths have been defined for you, to show the model behaves. (Use the case buttons at the top of the Animation window.)
You should look at the model in the Model window to see how each segment is calculated.

You have

Seen how minimising the quantity called the action can lead to a parabola.
This works for a ball, starting in one place, and ending up in another, travelling in a uniform gravitational field.
You calculate this for each segment, then sum the contribution of each segment.

Files provided

leastaction.mdl

Getting it to work

This is very much for the interested student. This is well beyond the level required by Advancing Physics, and yet the essential idea is quite simple, and the chance to manipulate a model makes it more approachable for many than simply reading about the idea.

Alternative approaches

Reading from Feynman, Lectures in Physics, volume 2, chapter 19

Social and Human context

Chapter 9 makes much of step by step calculations involving local causes, but here we do not look for mechanism at all – just a global minimum. Which is the 'truer' account of how the world works?

Safety

nil