# Bandwidth: a further teaching note

These notes discuss the bandwidths of analogue and digital signals, and the relationship of the information carrying capacity of a channel to bandwidth and noise. The discussion is not a rigorous derivation of the various relationships, but stresses the essential information and physical processes involved.
Some frequently asked questions raised by students of *Advancing Physics* are answered.
The notes are based on the excellent discussion in J C G Lesurf 'Information and measurement', Institute of Physics Publishing, 1995, which is well worth having on the library shelves.

## Bandwidth of analogue signals and bandwidth of a channel

**Question:**
"You say that analogue telephone signals are limited to frequencies up to about 4 kHz. Is this because the telephone lines are limited to that capacity, or is it to get more calls on the line, with 4 kHz being enough because that is about the highest component frequency in the human voice?"

**Answer:**
The bandwidth assigned to telephone calls is a *design decision*. The range of frequencies allowed is filtered to be in the range 300 Hz to 3400 Hz (bandwidth of the signal a bit less than 4 kHz). This is so as to be able to pack as many calls as possible on long distance broad-band lines. One way to do this is to assign each call a band 4 kHz wide on the higher range of frequencies carried by the broadband line (this is called frequency multiplexing). The idea is basically the same as the way radio stations are spaced out along, for example, the FM band.

However, the design decision has also to take into account the range of frequencies needed to reproduce speech sufficiently faithfully. Experiments with a frequency analyser will show that speech contains frequencies outside the range 300 Hz to 4 kHz, but that filtering out all but this range leaves the speech generally recognisable and intelligible. But as you know, it isn't always easy to recognise someone's voice on the 'phone, and it is sometimes hard to dictate single letters (as in a post code) so that they are clear (e.g. did you say 'B' or 'P'?). So the limitation of the telephone bandwidth is sometimes noticeable.

The telephone line from your house to the nearest exchange has only to carry one call at a time. So it is kept simple, using twisted pairs of wires. There is no need for this line to carry analogue signals at more than about 4 kHz, so this part of the telephone system is 'designed down' to a relatively low channel bandwidth. This is the reason new high capacity lines have to be brought to your house if you want high bandwidth digital communications from home. It is the

reason why cable TV is brought into your house by a special high bandwidth cable, not via telephone lines.
In fact however, the actual bandwidth possible using the ‘local loop’ of simple pairs of wires is quite a bit larger than 4 kHz. The lines are capable of carrying the 64 kbits per second required by ISDN (Integrated Services Digital Network) for home use (a pair of such circuits are provided with one ISDN connection). But these lines must now not be connected through the filters which otherwise limit the bandwidth to the range 300 Hz to 3400 Hz. So the equipment at each end of the lines has to be changed: only the wires stay the same. For business users, new coaxial cable or fibre optic cables are used, completely replacing the simple copper wire pairs. Thus the roughly 4 kHz bandwidth of the ‘local loop’ of lines from homes to telephone exchange is an important limitation on the development of broadband communication at home. It is one reason why the internet can be slow.

**Summary of terms:**
Just to define terms and be clear about the difference between signal bandwidth and channel capacity:

**Bandwidth of analogue signal** The spectrum of an analogue signal contains a range of frequencies. The range, *B = maximum frequency – minimum frequency*, is the bandwidth of the analogue signal. In many cases, this is in effect equal to the maximum frequency (the signal can extend down to dc).

**Bandwidth of a channel** A signal has to be sent along a communication channel, for example wires. The bandwidth of a channel is the range of frequencies it can carry. Often, this is the same as the maximum frequency it can carry. Fundamentally, this is determined by the fastest response time of the channel to a change in input voltage.
To transmit an analogue signal across a channel, the channel must have a bandwidth greater than or at least equal to the signal bandwidth. See below, however, about the effects of noise.

## Computers, modems and internet communications

**Question:**
“My modem will transmit and receive down the telephone line at up to 56 kbits per second. Then how can it be true that the telephone line is limited to frequencies up to about 4 kHz, which I think means up to 8 kbits per second?”

**Answer:**
Your argument is mixing up sending bits and sending an audio signal. If the bits in the data on your computer were sent directly down the normal bandwidth limited telephone line as electrical pulses, then the highest rate would indeed be 8 kbits per second. A long series of 1’s and 0’s sent at this rate would sound like a 4 kHz whistle.

But this is not how it is done. An audio signal of 4 kHz bandwidth can transmit properly encoded information at considerably more than 8 kbits per second. The modem (full name ‘modulator-demodulator’) takes the bit stream from your computer and encodes it as an analogue audio signal.

It isn’t at all obvious that the rate of transmission of information by an analogue signal can be calculated in bits, which seem to be a purely digital concept. The way to do it is to think about converting the analogue signal to digital form, in a way that captures all the information the analogue signal carries. Then the rate of production of bits is equal to the rate of arrival of information carried by the analogue signal.
The calculation is best thought of in two steps. First, the analogue signal can be turned into samples taken at twice the maximum audio frequency. In the case of a signal limited to 4 kHz this is 8000 samples per second. At this rate of sampling, nothing is lost. No further information is gained by sampling more frequently.
*Sampling rate* = 2*B* (where *B* is audio signal bandwidth).

Second, digitise each sample into digital words each *b* bits long. Then:
*Rate of transfer of information I = sampling rate* $\times$ *bits per sample*
*rate of transfer of information in bits per second I* = 2*Bb*.
It looks as though you can increase the rate at which bits are picked up by increasing the resolution 1 in $2^b$ with which each sample is measured. Not so. Any channel introduces noise. If the noise varies by (say) 1000 of the signal variation, there is no point in measuring a sample to better than 1 in 1000. This would make *b* = 10 bits the largest feasible value of *b* (since $2^{10}$ is approximately 1000). If the ratio of noise voltage variation $V_N$ to signal variation $V_S$ is small, the largest appropriate bit-length of a sample is given by $1/2^b = V_N/V_S$. That is, $b = \log_2 (V_S/V_N)$ approximately.

Thus in this approximation:
*rate of transfer of information in bits per second I* = 2*Bb* = $2B \log_2 (V_S/V_N)$.
It is common to express this in terms of signal and noise powers, *S* and *N*, proportional to the voltage variations squared. In this case, again approximately:
*rate of transfer of information in bits per second I* = 2*Bb* =
$2B\log_2 \sqrt{S/N} = B\log_2 (S/N)$
Telephone lines can be quite noisy. If the signal to noise power ratio were 1000 to 1, this provides communication by the analogue channel at about 10*B* in bits per second, or 40 kbits per second for a 4 kHz channel.

In practice, a telephone line will be noisier than that, and the bit rate achievable on an analogue signal is substantially less. Yet modems regularly achieve up to and above 40 kbits per second. They do this by also compressing the bit sequence from the computer. One simple way is run-length compression. A run

of 1's, or of 0's, is sent simply as the number of 1's or 0's in the run. You may have downloaded compressed music files on the internet.
[A modem is also clever about how the audio signal is modulated to encode the compressed digital signal, using both amplitude and frequency modulation simultaneously. Phase modulation may also be used.]

**Summary:**
The argument is easy to misunderstand. It is **not** about how quickly digital bits can be sent along a line as 0's and 1's. It is about how rapidly an audio signal carries information, **measured in** (not transmitted as) bits per second. Indeed, if an audio signal were turned into a digital stream of bits at source, and sent along a line as a bit stream, the bandwidth of the channel required would be **larger** than the bandwidth of the analogue signal (because of the need to send *b* bits per sample). That is exactly why digital mobile 'phones (and digital television) eat up bandwidth. The reverse process, that is, taking a digital source and sending it at a bit rate *I*, encoded as an analogue signal, requires an analogue bandwidth of **less than** the bit rate, by a factor depending on the signal to noise ratio of the channel.

**Exact expression for channel capacity** The argument above neglects the fact that the signal sampled is the total voltage variation, signal plus noise. It is also necessary to take into account that the signal and noise are uncorrelated, so that the square of the total voltage variation is the sum of the squares of the signal and noise variations. Then the exact expression for the maximum bit rate for an analogue channel of bandwidth B and signal to noise power ratio *S/N* becomes:
*rate of transfer of information in bits per second I* = $B\log_2(1+S/N)$
This is Shannon's theorem.

Notice that this exact expression correctly gives the rate of transfer of information as tending to zero when the signal power becomes much smaller than the noise power. In this case *S/N* tends to zero, and $\log_2(1+S/N)$ tends to $\log_2(1)=0$.

## Bandwidth as valuable real estate
Higher and higher frequencies are being used to transmit signals. For example, a microwave link may operate at 30 GHz.

**Question:**
So does that mean that the bandwidth available is equal to the maximum frequency, that is 30 GHz?

**Answer:**
No. Or rather, it would be if digital signals were sent by switching the carrier on and off like an Aldis lamp at up to 30 GHz, but that would interfere with all other communications in the whole frequency range. Instead, a range of frequencies near 30 GHz, say 29.5 GHz to 30.5 GHz is used as a channel 1 GHz wide. Only by using high frequencies can a narrow band near that frequency have the wide

bandwidth needed for, for example, digital television. And even then the 1 GHz bandwidth is divided into around 100 channels about 10 MHz wide, each sufficient to send suitably compressed digital television signals.

## Further background ideas

### Transmitting a signal

A signal consists of changes of voltage presented to the channel. Suppose that it takes the channel one microsecond to respond to a change of input voltage. Samples can therefore not be sent faster than 1 MHz. However, the changes of voltage over two samples will look like one up and one down (or one down and one up); that is, one cycle of a frequency of 0.5 MHz, so that the maximum frequency of the voltage waveform carrying the samples is half the sampling rate. This makes it reasonable to expect the sampling rate to be double the channel bandwidth (the sampling theorem).

### Sampling an analogue signal

An analogue signal can be translated into a sequence of binary numbers. If a total of $N$ samples each given as a $b$-bit binary number is taken, the amount of information in bits carried by the signal is $Nb$. If the samples are taken at the rate $f$, the rate of transmission of information in bits per second is $fb$.

### Reconstructing a signal from a sample

An analogue signal can be completely reconstructed from or taken apart into samples taken at twice the highest frequency in the signal (Sampling Theorem). The two contain exactly the same information. The factor two comes from the fact that each Fourier component of the spectrum has an amplitude and a phase, requiring **two** numbers to specify each component. Given the Fourier components, the signal can be exactly reconstructed: the two representations (time and frequency domain) contain exactly the same information.

### Digital signals need a lot of bandwidth

One way to transmit a sampled signal is to send it as a sequence of bits, sent at the rate of at least $fb$, where $f$ is the number of samples per second, and there are $b$ bits per sample. The worst case is where bits alternate ones and zeros. A one followed by a zero looks like one cycle of the transmitting frequency. Thus, on this argument, the channel must be able to carry frequencies up to at least half the bit-rate.

Sending signals digitally rather than as analogue signals increases the bandwidth needed. An analogue signal of frequency $f$ requires a bandwidth of the order $f$ (neglecting noise). A digital bit sequence of $b$-bit numbers representing each sample representing the same signal requires a bandwidth of order $fb$.

### Why noise makes a difference

Suppose that a signal varying in the range ± 1 V is sent along a channel which generates random noise in the range ± 1 mV. It is not worth using more than 10 bits per sample, because this already divides the voltage range into 1024 levels, each comparable to the 1 mV random noise variation present. More bits per sample would merely transmit the noise component of the signal more faithfully. However, limiting the number of bits per sample limits the rate of transfer of information. Thus the higher the signal to noise ratio, the faster information can meaningfully be sent. The capacity of a channel, the rate at which it can be used to transmit information, therefore depends on the signal to noise ratio as well as on its bandwidth.

**Noise increases channel bandwidth required**
Noise increases the number of errors made. To correct for errors, more information must be sent so that the true signal can be computed from the 'redundant' extra information. This increases the required bit rate. Thus it is sensible to ignore any small numerical factors, and take the minimum signal bit-rate as a guide to the order of magnitude of the channel bandwidth required. In practice, the required bandwidth to transmit a signal as a digital bit-sequence may be several times the minimum signal bit-rate, not half the bit-rate.

**Regularities in signals reduce the bandwidth needed**
When signals are coded digitally, it is often easy to **reduce** the required bit rate, and so the bandwidth required. For example, in sending digital TV signals, it is necessary only to send the values of pixels, which change from one frame to the next. This, and other kinds of data compression, can reduce the required bandwidth well below that given by a simple calculation (e.g. from numbers of pixels, pixels per second, and bits per pixel).

Another example is the use of a digital mobile telephone. Given speech signals up to say 4 kHz which would match the analogue telephone circuit bandwidth, it looks at first as though sampling at 8 kHz is needed, which with 8 bits per sample would require an information transmission rate of 64 kHz. But in fact a digital telephone can 'squeeze' the bit transmission rate, by compressing the signal. It is especially helpful that telephone conversations actually contain quite a lot of silence. It's cheaper in bits to send a message "nothing for 100 ms" than to send 6400 zeroes.

## Orders of magnitude for bandwidth

**Question:**
So just how much channel bandwidth is needed for a given signal?

**Answer:**

An analogue signal of signal bandwidth *B* needs a channel of bandwidth of the order of magnitude *B*. It can be sent as samples at the rate 2*B*, but each sample looks like half a cycle of a signal.
A digital bit stream sending binary digits at the rate *I* in bits per second needs a bandwidth of the order of magnitude *I.*
An analogue signal of bandwidth *B*, digitised and sent as a bit-stream, needs a bandwidth of the order of magnitude $I = Bb$, where samples are digitised as *b*-bit words. The higher the signal to noise ratio, the larger the *b*-bit words can be, measuring the samples with higher resolution.
These estimates are all very approximate, never better than to a factor 2 and sometimes to a factor 10. They do not allow for the effect of noise on the channel, nor for the advantages of data compression.
The bandwidth required can be decreased by data compression, often by a factor of 10 or more.

Noise on a channel decreases the rate at which information can reliably be sent, or alternatively increases the bandwidth required to send information at a given rate. There is a trade-off between channel bandwidth and signal to noise ratio, for a given rate of transfer of information. FHM radio and mobile telephones keep signal powers low by using wide bandwidths.