2nd section A test for Clockwork Universe: Time 30 minutes

1. Choose the correct value from the list below for the energy stored in a 100 μF capacitor with a p.d. of 25 V across it.

0.0025 J 0.031 J 0.063 J 2.5 J

(Use ½CV^2 or ½QV) energy = ……0.031.J ✓

[1]

2. The graph in Fig. 1 shows the decay curve for a sample of radioactive nuclei decaying to form stable nuclei.

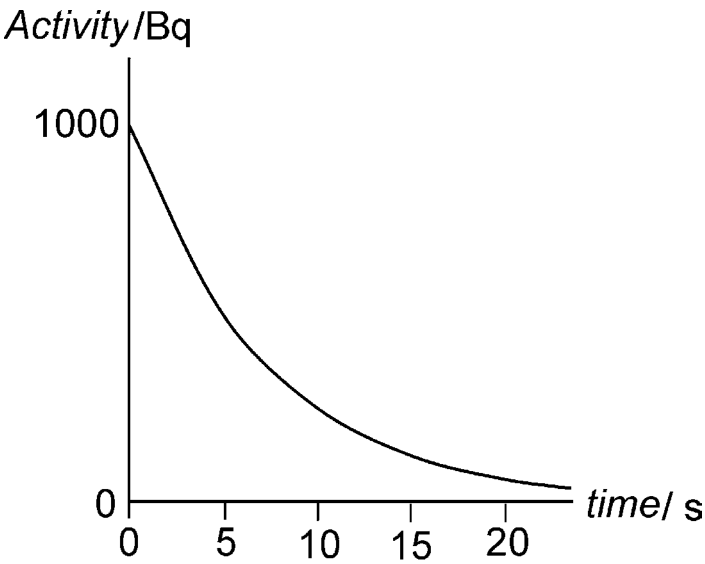

Fig. 1

(a) Show that the decay constant λ is about 0.14 s^{-1}.

Half life is about 5 s ✓ so λ = (ln 2)/5 s = 0.139 s^{-1} ≈ 0.14 s^{-1} ✓

[2]

(b) Show that the sample originally contained about 7000 radioactive nuclei.

$\frac{dN}{dt} = -\lambda N$ so 1000 Bq = 0.14 N so N = 1000/0.14 = 7140 ≈ 7000 ✓ m ✓ e

[2]

3. The graph in Fig. 3 shows the displacement-time graph for a 0.50 kg mass attached to a spring.

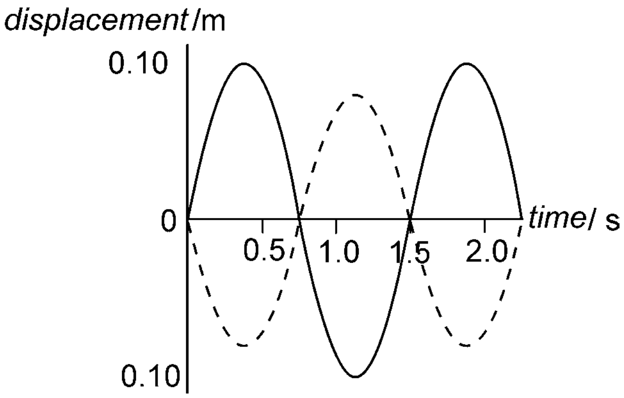

Fig. 3

(a) State the period of oscillation of the mass-spring system.

period = 1.5 s ✓ [1]

(b) Calculate the spring constant k of the spring.

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ so } 1.5\text{ s} = 2\pi\sqrt{\frac{0.50\text{ kg}}{k}} \Rightarrow \left(\frac{1.5\text{ s}}{2\pi}\right)^2 = \frac{0.50\text{ kg}}{k}$$

$$0.057\text{ s}^2 = \frac{0.50\text{ kg}}{k} \Rightarrow k = \frac{0.50\text{ kg}}{0.057\text{ s}^2} = 8.8\text{ N kg}^{-1}$$ ✓ m

k = 8.8 ✓ e unit N kg^{-1} (or kg s^{-2}) ✓ [3]

(c) Without calculating any values, sketch on Fig. 3 above the acceleration-time graph for this oscillation.

Same T, cuts y-axis at the same points ✓ upside down ✓ (ticks go on graph)

[2]

4. A 2200 W kettle contains 2.5 kg of water. Calculate the rate at which the temperature rises.

$c = 4200$ J kg^{-1} K^{-1}

$$\Delta E = mc\Delta T \text{ so } \frac{\Delta E}{\Delta t} = mc\frac{\Delta T}{\Delta t} \text{ i.e. } P = mc(\text{rate at which the temperature rises})$$

$$\text{so rate } = \frac{P}{mc} = \frac{2200\text{ W}}{2.5\text{ kg} \times 4200\text{ J kg}^{-1}\text{ K}^{-1}} = 0.21\text{ K s}^{-1}$$ ✓ m ✓ e

[2]

5. Fig. 4 shows gravitational equipotentials near a binary star system consisting of two stars A and B.

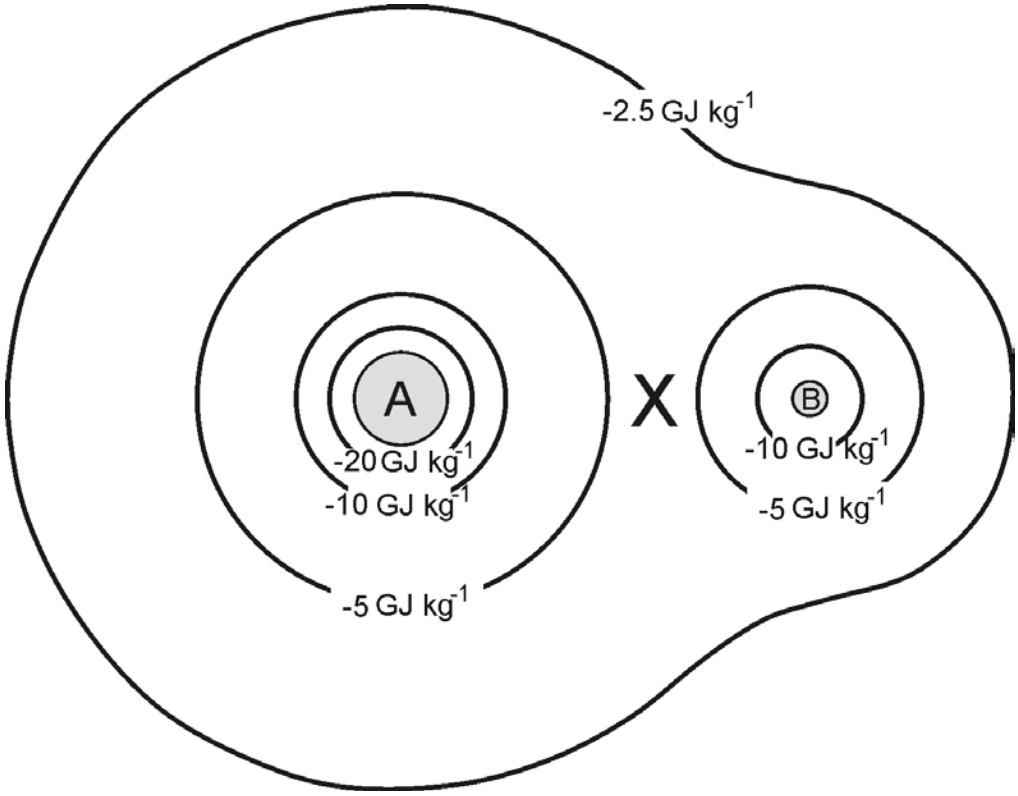

Fig. 4

(a) Explain how the gravitational equipotentials show that the gravitational field strength becomes weaker as you move away from each star.

$g = \frac{dV}{dx}$ and the increments of V, ΔV, are getting smaller while the corresponding values of Δx are getting greater, so g is getting less on both counts.
✓ for identifying relationship between V and g, ✓ for using either variation to explain the drop in g

[2]

(b) Estimate the energy needed to move a 100 kg mass from the surface of star A to a very great distance away.
V_{grav} at the surface of A is about -30 GJ kg^{-1} (allow between -20 and -40 GJ kg^{-1}). ✓

So energy per kg to go from surface to far away (where $V_{grav} = 0$) is 30 GJ ✓
Thus the energy needed for 100 kg = 100 kg × 30 × 10^9 J kg^{-1} = 3 × 10^{12} J ✓e

[3]

(c) Mark with an X the point on the diagram where the resultant gravitational field strength is 0.
Should be on the line joining A and B, between the two -5 GJ kg^{-1} equipotentials (draw a V-distance graph along this line to convince yourself!) ✓ (tick goes on graph)

[1]

6. Calculate the centripetal acceleration of the Earth as it orbits the Sun.

radius of Earth's orbit = 1.5×10^{11} m
1 year = 3.2×10^{7} s

$$v = \frac{2\pi R}{T} = \frac{2\pi \times 1.5\times10^{11}\ \text{m}}{3.2\times10^{7}\ \text{s}} = 2.9\times10^{4}\ \text{m s}^{-1}\ \checkmark$$

$$a = \frac{v^2}{R} = \frac{(2.9\times10^{4}\ \text{m s}^{-1})^2}{1.5\times10^{11}\ \text{m}} = 5.8\times10^{-3}\ \text{m s}^{-2}\ \checkmark\ \text{m}\ \checkmark\ \text{e}$$

[3]

7. 0.50 mol of an ideal gas at 350 K occupy a volume of 0.40 m^3.

(a) Show that the pressure is about 3600 Pa.
R = 8.3 J $K^{-1}mol^{-1}$

$$pV = nRT \Rightarrow p = \frac{nRT}{V} = \frac{0.50\ \text{mol}\times 8.3\ \text{J K}^{-1}\ \text{mol}^{-1}\times 350\ \text{K}}{0.40\ \text{m}^3} = 3630\ \text{Pa} \approx 3600\ \text{Pa}$$

✓ m ✓ e

[2]

(b) Calculate the new pressure if the temperature drops to 290 K, the volume remaining constant.

You can recalculate with $pV = nRT$, but direct proportion is easier:

$$p \propto T\ \text{so}\ \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = \frac{p_1}{T_1}\times T_2 = \frac{3600\ \text{Pa}}{350\ \text{K}}\times 290\ \text{K} = 3000\ \text{Pa}\ \checkmark\ \text{m}\ \checkmark\ \text{e}$$

[2]

8. In a hydrogen atom, the lowest possible energy state is 1.6×10^{-18} J below the next higher energy state.

(a) Show that the Boltzmann factor for these two energy states at a temperature of 6000 K is about 5×10^{-9}.
$k = 1.4 \times 10^{-23}$ J K^{-1}

$$\text{Boltzmann factor} = e^{-\frac{E}{kt}} = \exp\left(-\frac{1.6\times10^{-18}\ \text{J}}{1.4\times10^{-23}\ \text{J K}^{-1}\times 6000\ \text{K}}\right)$$

$= 5.3 \times 10^{-9} \approx 5 \times 10^{-9}$ ✓ s ✓ e

[2]

(b) Explain why, at a temperature of 6000 K, only about 1 hydrogen atom in 2×10^{8} will be in the higher of these two states.

5×10^{-9} is the proportion of those in the higher state ✓ so if there are 2×10^{8} in total, you expect $2 \times 10^{8} \times 5 \times 10^{-9} = 1$ in the higher state. ✓

[2]