Chapter 14
Short answer question

This question is about how the density of air varies with the height above the Earth's surface.

At sea level the density of the atmosphere is about 1.2kg m^{-3}.

Consider nitrogen at a temperature of 300K.

(a) Show that the average energy of a particle at this temperature is about 4.1×10^{-21} J.

$k \times T = 1.38 \times 10^{-23} \times 300$

$= 4.14 \times 10^{-21}$ J

$\approx 4.1 \times 10^{-21}$ J

(b) Show that the energy *E* required to lift a nitrogen molecule of a mass 4.6×10^{-26} kg to a height 3000m above sea level is about 1.4×10^{-21} J.

$E = mgh = 4.6 \times 10^{-26} \times 9.8 \times 3000$

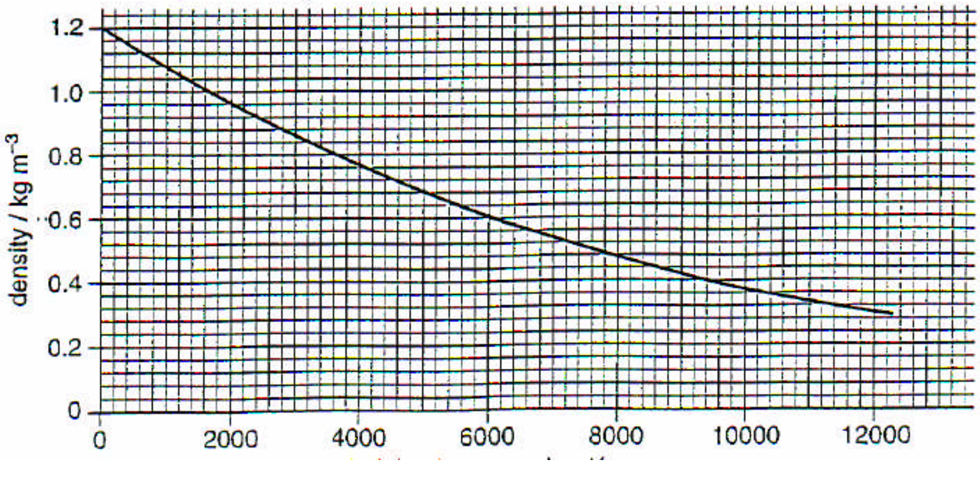
$= 1.35 \times 10^{-21}$

$\approx 1.4 \times 10^{-21}$ J

(c) Show that the Boltzmann factor $e^{-E/kT}$ for the energy *E* found in part (b) at a temperature of 300 K is about 0.34.

$$\text{B. factor} = e^{-1.4 \times 10^{-21} / 1.38 \times 10^{-23} \times 300} = 0.34$$

The graph below shows density of air against height above sea level. The atmospheric pressure is assumed to remain constant with height in this model.



(d)

(i) The graph shows that the density decreases with increasing height. Explain how the Boltzmann factor helps to account for this fact

Boltzmann factor measures the numbers of particles at energy levels of energy difference E.

(ii) Propose and carry out a test to decide whether the density falls exponentially with height above sea level.

Proposed test: Constant ratio test

Calculation:

$$\frac{\text{Density }1}{\text{Density }2} = \frac{\text{Density }2}{\text{Density }3} \text{ etc.}$$

Conclusion: above shows exponential

(e) In fact, atmospheric pressure decreases with height above sea level. Explain why this makes the Boltzmann factor became smaller with height above sea level.

kT is smaller

$\frac{E}{kT}$ is bigger

$\frac{I}{e^{\frac{E}{kT}}}$ is smaller

$e^{-\frac{E}{kT}}$ is smaller