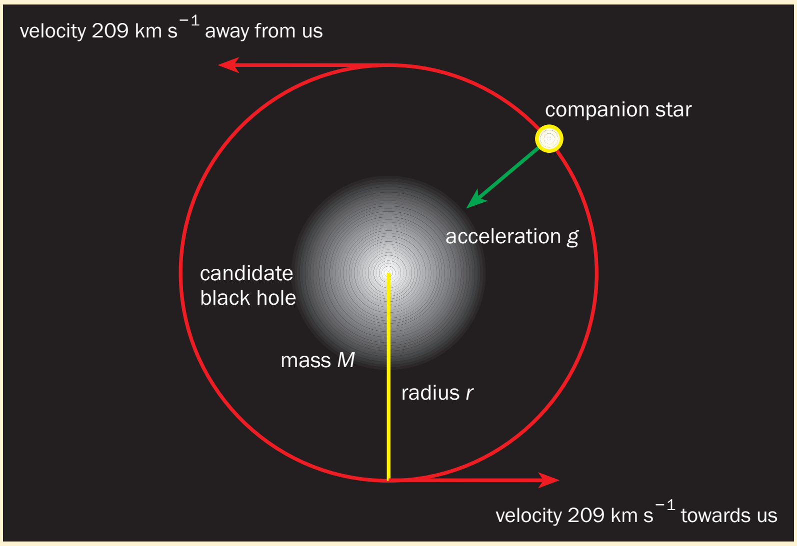
ladder of distances, each new rung of the ladder being calibrated by lower rungs with which it overlapped. Each side thought the other was making unjustifiable guesses and assumptions; many harsh words were spoken. For a decade or so, astronomers had two competing scales. Only very recently has there been a better measure of agreement.

'Weighing' the Universe

Armed with Doppler shift measurements of velocities it becomes possible to measure the masses of stars, black holes, even galaxies. Here's an example.

Ever since the idea of gravitational 'black holes' was suggested, astronomers have looked for possible traces of black holes, even though a black hole is itself invisible. But an invisible object can be 'seen'

Measuring the mass of a black hole



acceleration $g = \frac{v^2}{r}$

acceleration measures the gravitational field of the black hole

field $g = \frac{GM}{r^2}$

$\frac{GM}{r^2} = \frac{v^2}{r}$

mass from v and r $\quad M = \frac{v^2 r}{G}$

Example: mass of a candidate black hole V404 Cygnus

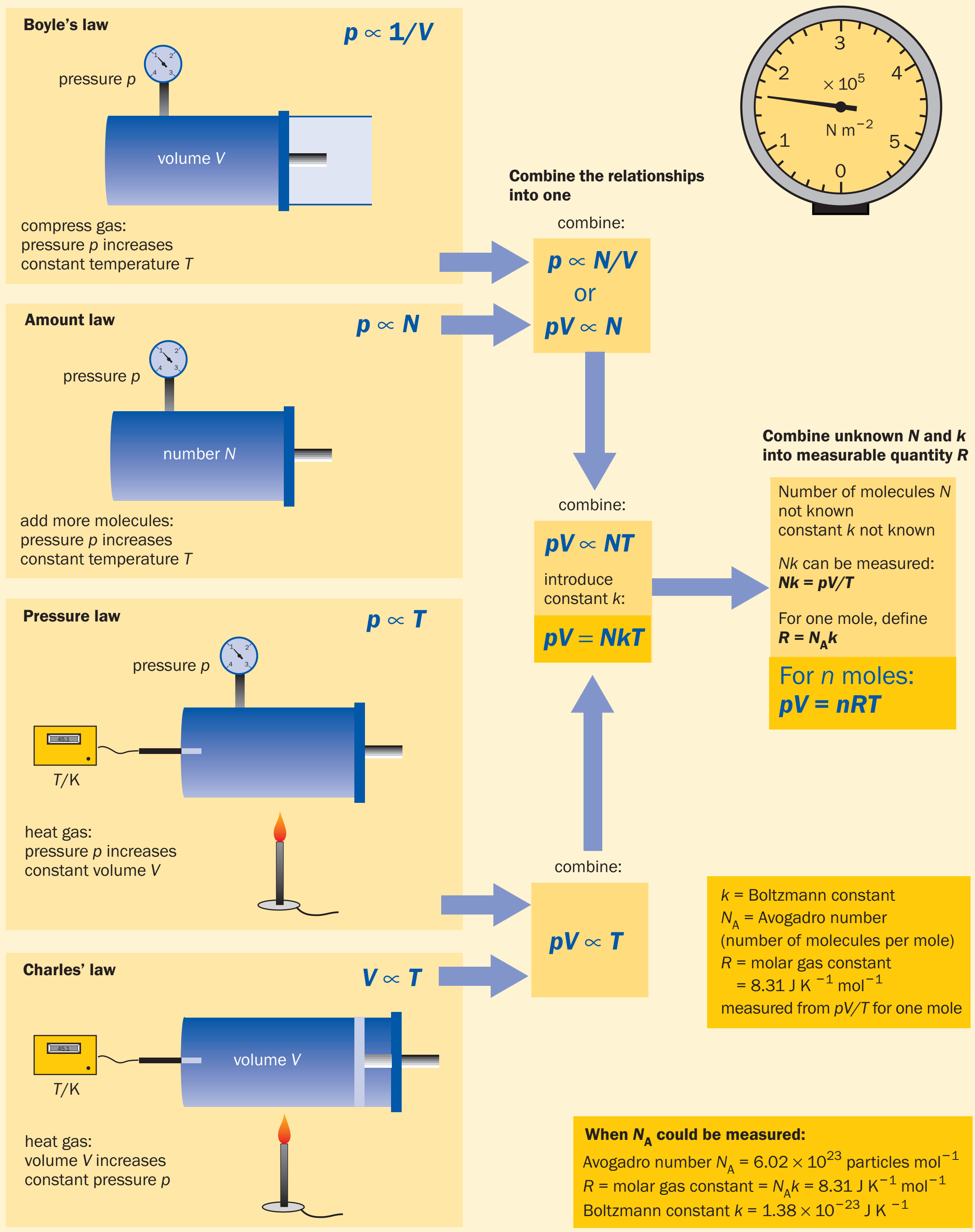
V404 Cygnus emits x-rays perhaps due to matter from an ordinary star 'falling into' a massive black hole companion which it orbits. Doppler shifts of light from the ordinary star show its velocity varying by plus or minus 209 km s^{-1}, over a period of 6.5 days.

speed *v* in orbit	from Doppler measurements, assuming orbital plane is in the line of sight	$v = 209$ km s^{-1}
radius *r* of orbit	from time of orbit and speed of star: star travels $2\pi r$ in 6.5 days at 209 km s^{-1}	$r = 18.7 \times 10^6$ km
acceleration *a* towards black hole	acceleration $= \frac{v^2}{r} = \frac{(209 \times 10^3 \text{ m s}^{-1})^2}{18.7 \times 10^9 \text{ m}}$	$a = 2.34$ m s^{-2}
		same quantity
gravitational field *g* of black hole	gravitational field = acceleration	$g = 2.34$ N kg^{-1}
mass *M* of black hole	from gravitational inverse square law, field $g = \frac{GM}{r^2}$	
	$M = \frac{gr^2}{G} = \frac{2.34 \text{ N kg}^{-1} \times (18.7 \times 10^9 \text{ m})^2}{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}}$	$M = 1.2 \times 10^{31}$ kg

mass of Sun = 2×10^{30} kg

mass M = 6 times mass of Sun

One law for all gases



- Exploring the rules for pressure
- Exploring the rules for volume

Heating in the home

Quite a bit of the energy used to keep a home warm goes to heating up the air, which has to change a few times an hour. But over the year heating water costs an ordinary family more than all the other heating they do—unless they live in a large and very badly insulated house in a cold climate. It takes about 20 MJ of energy to provide a hot bath. At 7p a kilowatt-hour (3.6 MJ) that costs about 40p. It's one reason to prefer a shower, which uses less hot water and therefore costs less.

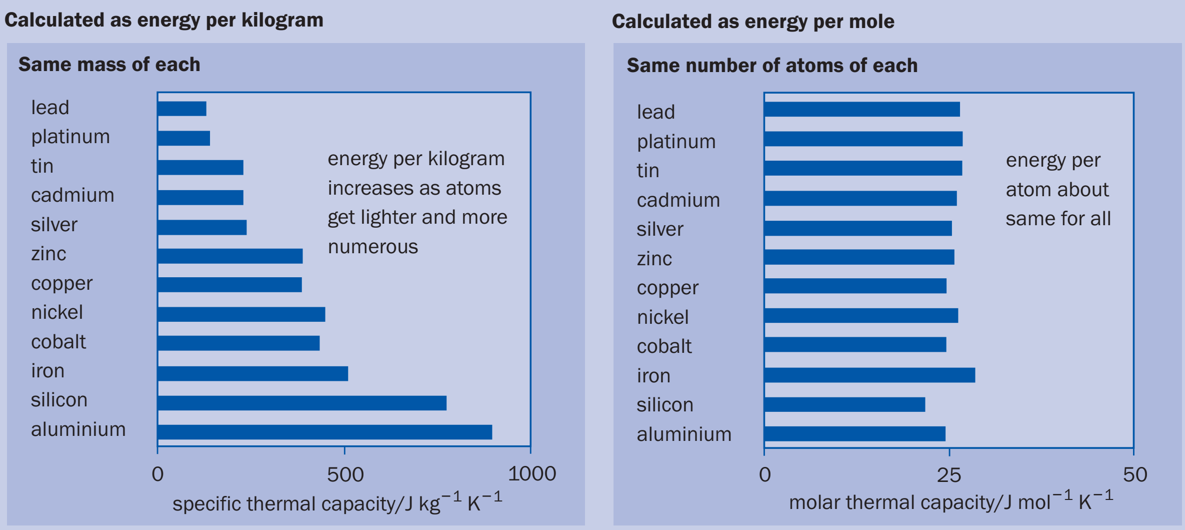
See the chapter 'Advances in Physics' for more about heating buildings.

Particles have energy roughly equal to *kT*

You certainly heat water for a bath by the litre—in effect by the kilogram. So the value of its specific thermal capacity expressed in joules per kilogram per kelvin makes good practical sense. But the kinetic theory of matter says that things may be simpler if thought about molecule by molecule. For ideal gases whose molecules just whizz about and don't spin or vibrate, the kinetic theory says that the energy per particle is just $\frac{3}{2}kT$.

More important, the kinetic theory can be stretched to cover all matter, not just gases, in an interesting if very rough way. The energy of thermal agitation per particle inside *any* kind of matter is some fairly small multiple of the quantity kT. What that multiple is varies from one kind of stuff to another, and may change a bit with temperature. But if you just calculate kT, you are within striking distance of the energy each particle possesses.

Thermal capacities of some metals



Monatomic metal elements have very similar thermal capacities per atom

On a nice summer day (27 °C, 300 K) the energy kT amounts to:

$$kT = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 300\,\mathrm{K} = 4.1 \times 10^{-21}\,\mathrm{J}$$

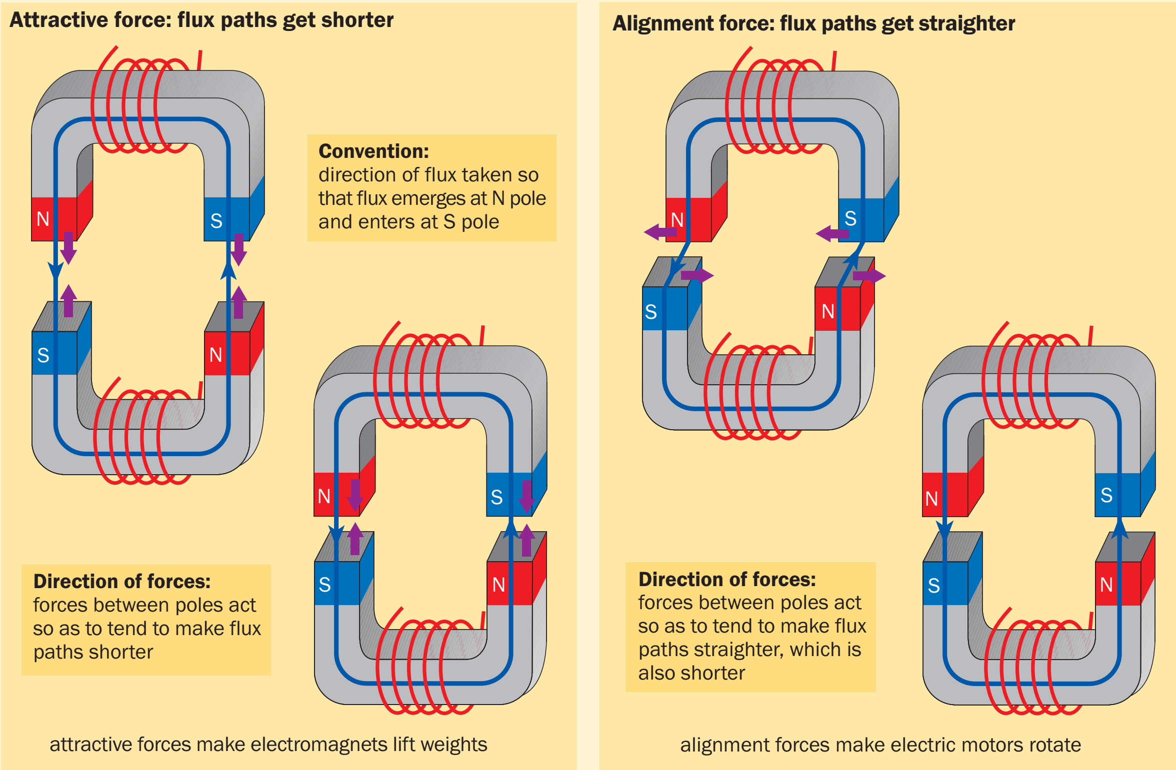
Compare this energy with something else. An electron moving through a potential difference of 1 volt changes energy by 1.6×10^{-19} J, an amount called 1 electron volt (1 eV). The random energy kT of thermal agitation is about 1/40 of an electron volt, 1/60 of the energy an electron gets from a 1.5 V torch battery.

So here is a general but very approximate rule:

particles in matter at temperature T each have energy of the order of kT

In chapter 14 you can see how this idea can be taken much further.

Flux and forces



Forces between poles are such as to tend to make flux paths shorter and straighter. The flux behaves like an elastic string

bound handwritten copies of Davy's lectures at the Royal Institution, Faraday was appointed to a post there. Starting as a bottle washer, he rose to become its head.

Faraday had a powerful faith that somehow all the forces of nature must be interdependent. He belonged to a tiny fundamentalist religious sect, the Sandemanians, who believed that religious truth was to be found in a simple honest consensual reading of the Bible. The Sandemanians saw the working of the whole physical world as an expression of the powers of one God. So Faraday saw science as 'reading the book of nature' and as a way to understand its unity. And throughout his life he sought unifying connections between light, electricity, magnetism and gravity.

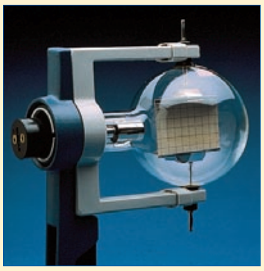
Over almost a decade, Faraday tried experiments in 'the hope of obtaining electricity from ordinary magnetism'. In 1831, Faraday found how to get measurable electric currents from magnetic fields. Joseph Henry found the same, at almost the same time.

Faraday wound coils of many turns on an iron ring. His wire wasn't insulated, so he used twine to keep the turns apart and put calico cotton fabric between the layers. One coil was connected to a battery. The other was joined into a complete circuit by a wire passing over a magnetic needle some feet from the ring. He found a remarkable effect. Whenever the battery was connected or disconnected, the magnetic needle deflected momentarily, showing a brief flow of current in the second coil. A steady flow of current had no effect, but a *changing* current did. Faraday made the modest note that this was 'very satisfactory'.

The experiment was the key to the connection Faraday had searched for all that time. Steady magnetic flux in the iron did not produce electric current, but *changing* magnetic flux did. The current was tiny, but it was *there*. The rest of the story is one of how to make the current bigger, to get the powerful generators which now supply industry and your home twenty-four hours a day.

It took a further half century before useful generators and motors were available, with power on a scale to begin to rival steam. As a result, the century that had begun with the battery ended with the beginning particle can travel as fast as light. Nor is the potential difference needed especially large. Since the velocity is proportional to the square root of the potential difference, to multiply the speed by a factor of 4 the potential difference must increase by a factor $4^2 = 16$. So increasing the potential difference from 16 000 V to about 250 000 V looks like enough to reach the speed of light. But it isn't.

Calculating kinetic energy and speed



$V = 1000$ V | $V = 16\,000$ V

kinetic energy

$E_K = qV$

charge on electron $q = 1.6 \times 10^{-19}$ C

$E_K = 1.6 \times 10^{-19}\ \text{C} \times 1000\ \text{V}$ | $E_K = 1.6 \times 10^{-19}\ \text{C} \times 16\,000\ \text{V}$

$E_K = 1.6 \times 10^{-16}$ J | $E_K = 25.6 \times 10^{-16}$ J

speed

If $E_K = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E_K}{m}}$$

mass of electron $m = 9.1 \times 10^{-31}$ kg

$v = \sqrt{\dfrac{2 \times 1.6 \times 10^{-16}\ \text{J}}{9.1 \times 10^{-31}\ \text{kg}}}$ | $v = \sqrt{\dfrac{2 \times 25.6 \times 10^{-16}\ \text{J}}{9.1 \times 10^{-31}\ \text{kg}}}$

$v = 1.9 \times 10^7\ \text{m s}^{-1}$ | $v = 7.7 \times 10^7\ \text{m s}^{-1}$

speed of light $c = 3 \times 10^8\ \text{m s}^{-1}$

speed about 6% of speed of light | speed about 25% of speed of light

p.d. V increases × 16
speed v increases × 4 ?

Limit to speed: no limit to energy or momentum

The actual speed of the accelerated electrons can be measured just by timing short bunches of them as they travel a known distance. As the potential difference is increased, their speed is indeed found to approach but never exceed the speed of light. Potential differences of only a few million volts, easily achieved with a Van de Graaff machine, are enough to get very close to the speed of light.

You might think that this means that there is also a limit to the energy the electron can be given. Not so. The kinetic energy is still equal to the change in electrical potential energy qV. But the expressions $\frac{1}{2}mv^2$ for the kinetic energy and mv for the momentum are only approximations. They are accurate enough at low speeds but **not** at high speeds. Different equations, first found by Einstein in his theory of relativity, are needed. These new equations require mass to be redefined, so that the mass of a particle is just its energy when at rest, such that

$E_{rest} = mc^2$ where c is the speed of light.

The factor c^2 just converts mass in kilograms to energy in joules. Particle physicists often express the masses of particles in energy units, frequently electron volts. An electron has mass about 0.5 MeV in these units, for example. A proton has mass about 1 GeV (2000 times greater). With both sides of the equation in the same units, you can think simply of the mass as *being* the rest energy, $E_{rest} = m$.

Einstein's equations linked mass m, total energy E_{total} and momentum p. At speeds close to the speed of light, where the kinetic energy is much larger than the rest energy, the relation is approximately $E_{total} \cong pc$. For photons, with zero rest energy (mass) the relation is exact.

The point is that an accelerator can't increase the speed of particles above the speed of light, but it can increase their energy and momentum as much as you like. In a high-energy accelerator like LEP (Large Electron–Positron collider) at CERN electrons and positrons move at 99.999% of the speed of light. The real point of a particle accelerator is not the final speed of the particles but the energy and momentum they carry. When such particles collide, some of their energy can become the rest energy (mass) of new, more massive particles. New forms of matter get created in such collisions.

- The ultimate speed

$$E = \frac{F}{q} = -\frac{\Delta V}{\Delta x}$$

If the field is changing continuously then it is better to use the limiting form of this equation:

$$E = -\frac{dV}{dx}$$

Again, in words this says

electric field strength = – potential gradient

As we have just explained, the negative sign is there because the field and the force point *downhill*. And downhill is a negative potential gradient. So that is the positive direction of the field, from high to low potential.

When we defined electric field earlier (page 154), we said it was the force per unit charge, with units N C^{-1}. But this is not the easiest way to measure an electric field. To find the uniform electric field between a pair of conducting charged plates, the easy way is to put a voltmeter across the plates to measure the potential difference V, and to use a ruler to measure the distance d between the plates. The field is uniform, so the equipotentials are all equally spaced, and the gradient is the same right across the gap. It's just a straight ramp. This makes the expression for the magnitude of the potential gradient very simple indeed:

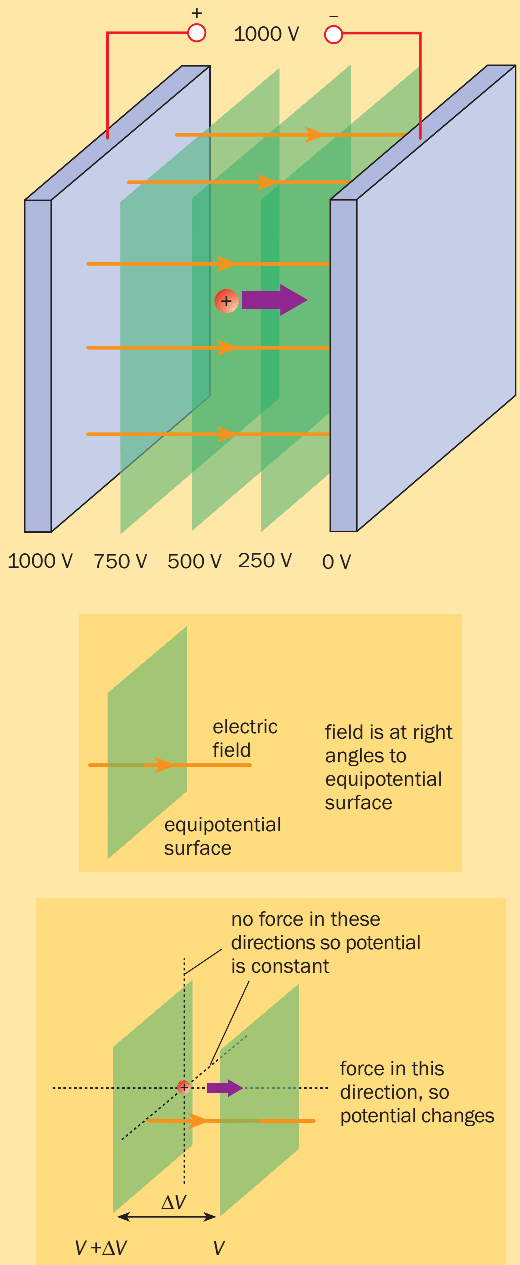
$$E = \frac{V}{d}$$

The units of potential gradient are volts per metre, V m^{-1}. The argument above shows that these units are the same as the other unit of electric field, N C^{-1}. They don't look the same, but they are just two ways of saying the same thing. You may recall something similar for the gravitational field (chapters 9 and 11). The units of gravitational field can be expressed as either N kg^{-1} or m s^{-2}.

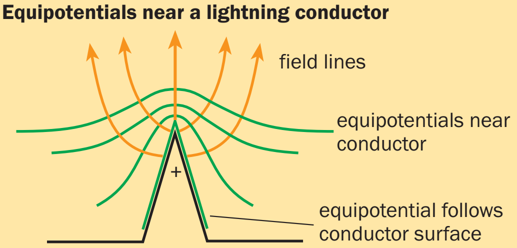
Large electric fields occur in Nature in lightning flashes, when potential differences of many millions of volts exist between parts of a thundercloud. Car engines use small lightning flashes to ignite the fuel–air mixture in the cylinder. In the spark plug a typical electric field is a few thousand volts per millimetre, that is, a few million volts per metre. Such electric fields are big enough to ionise the air, creating a spark. Some people like to use ionisers in the home, feeling that they improve air quality. Ionisers use intense electric fields to ionise air.

Field lines and equipotential surfaces

A uniform field



Equipotentials near a lightning conductor



Field lines are always perpendicular to equipotential surfaces

Questions

$e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.9 \times 10^{-12}$ C^2 N^{-1} m^{-1}, $c = 3 \times 10^8$ m s^{-1}, $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.7 \times 10^{-27}$ kg.

1 An electron in an oscilloscope is accelerated through a potential difference of 1 kV.

(a) Write down the kinetic energy in electron volts gained by the electron.

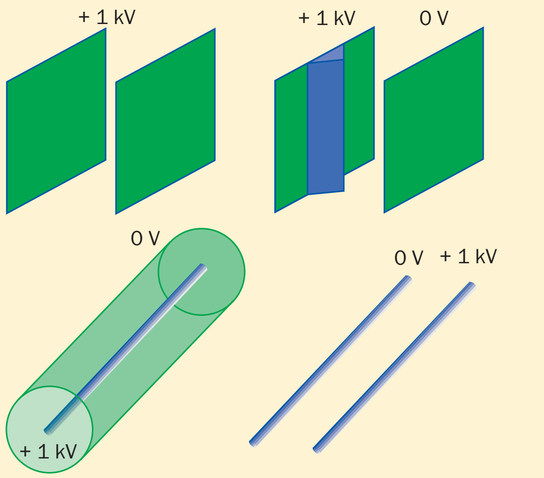
(b) Calculate the rest energy mc^2 of an electron in electron volts. Is the kinetic energy gained by the electron much smaller than its rest energy?

(c) The kinetic energy of a mass m moving at speed v is given by $\frac{1}{2}mv^2$ if the kinetic energy is much less than the rest energy. Show that for a charge q accelerated through potential difference V the speed v is given by $v = \sqrt{2qV/m}$

(d) The ratio q/m of charge to mass for electrons is 1.8×10^{11} C kg^{-1}. Calculate the speed of an electron accelerated through 1 kV.

(e) Protons are 1840 times more massive than electrons. Calculate the speed of a proton accelerated through 1 kV.

2 Sketch equipotentials and electric field lines for the sets of charged conducting electrodes shown.



3 Give one example each of the use of an electric field and a magnetic field in a particle accelerator. In each case make sure that you:

(a) describe the type of accelerator briefly,

(b) state the use to which the electric or magnetic field is put,

(c) indicate the shape of the electric or magnetic field,

(d) show the directions of any forces on an accelerated particle,

(e) state how to calculate the magnitude of forces on the accelerated particle.

4 Calculate the electric field, the electric potential and the force on an electron

(a) at a distance equal to the radius of a hydrogen atom (approximately 0.5×10^{-10} m) from a proton,

(b) at the distance equal to 10 times the radius of a proton (approximately 10^{-15} m) from a proton,

(c) at a point midway between the flat surfaces of the electrodes in a spark plug, spark gap 0.4 mm, potential difference 10 kV.

5 Fill in the missing elements in the relationships in the table below, which compares gravitational and electric field and potential.

Field and potential of spherical masses and charges

Gravitational field:	*Electric field:*
$g = -[\,]\dfrac{m}{r^2}$	$E = \dfrac{q}{4\pi\varepsilon_0 [\,]}$
Gravitational potential:	*Electric potential:*
$V_{grav} = -G\dfrac{m}{[\,]}$	$V_{elec} = \dfrac{q}{[\,]r}$

6 Electric and gravitational fields and potentials can be expressed in various units.

(a) Show from the definition of electric field strength that the units of electric field can be written as N C^{-1}.

(b) Show from the relation between field and potential gradient that the electric field can also have the units V m^{-1}.

(c) Use the fact that the unit V can be written as J C^{-1} to show that the unit V m^{-1} is the same as the unit N C^{-1}.

(d) If there were a 'gravitational volt' V_{grav}, its unit would be J kg^{-1}. What would be the unit of gravitational field, in terms of 'gravitational volts'?

7 An electron travels at low speed v at right angles to a uniform magnetic field B. The force on the electron has magnitude $F = evB$.

(a) In what direction is the force $F = evB$? Explain why this force is equal to the centripetal force mv^2/r.

(b) Show that the radius of curvature of the path of the electron is given by $r = p/eB$, where p is the momentum of the electron.

(c) Assume that the expression $r = p/eB$ in (b) is valid at all speeds. Find the radius of curvature of the path in a magnetic flux density $B = 0.1$ T of a relativistic electron of energy $E = 100$ MeV, for which the relationship $p = E/c$ is a good approximation.

So they could map the charged particles inside the proton or neutron.

The collision of a high energy electron with a quark does much more than just deflect the electron through a large angle, as in Rutherford's experiments. Instead, particle creation starts happening. Quark–antiquark pairs materialise out of the energy of the interaction. Following roughly the path of the quark as it is given a huge kick by the electron, they emerge as a 'jet' of new particles. Many of the new particles are mesons, which the theory says are quark–antiquark pairs. This kind of inelastic scattering, with energy going to create sprays of new particles, is obviously more complicated than elastic scattering.

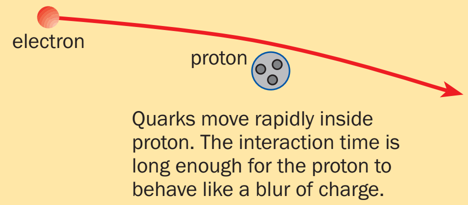
However, Richard Feynman realised that relativistic effects make this complicated situation much simpler. To the accelerated electron, the proton must look like a particle approaching it at nearly the speed of light. Time dilation will slow down the motion of the quarks so that they seem almost at rest. They become 'sitting targets'. Also length contraction shrinks the nucleon in the direction of approach, making it appear like a flat disc with three target pancakes inside it.

This smart thinking of Feynman's made it much easier to deduce the pattern of charges in the nucleon from the pattern of scattering. It was possible to show that the scattering was consistent with the existence of three particles inside a neutron or proton. From the way the amount of scattering depends on the charge on the scattering particle, it was possible to check that quarks do indeed have charges a fraction of the fundamental unit *e*.

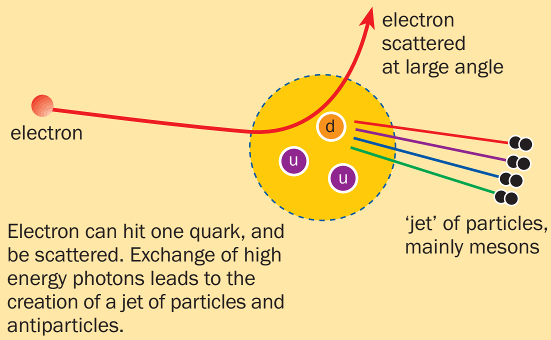
Feynman described the discovery of quarks by deep inelastic scattering as like 'studying a swarm of bees by radar'. Similarly, Rutherford once likened firing neutrons at nuclei to shooting at birds in the dark in a country where there aren't many birds! The discovery of the nucleus by elastic alpha scattering and the discovery of the quark by inelastic electron scattering show how a similar technique can yield crucial results on radically different scales of size.

Deep inelastic scattering

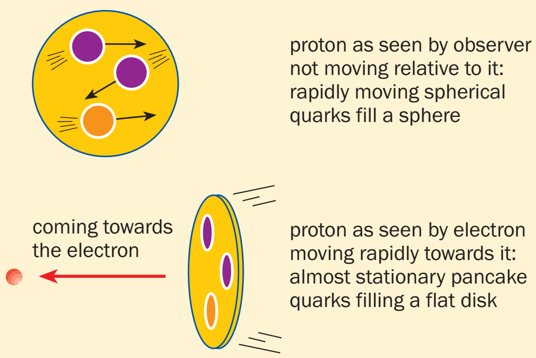
Medium energy: elastic scattering



High energy: deep inelastic scattering



Quarks as relativistic stationary pancakes



Energy and scale

You will have noticed that the smaller the scale you want to resolve, the larger the energy you need to probe it. This is the main reason why particle physics needs to build ever larger and more expensive accelerators.

The detector array in a large accelerator fills an underground hall as big as a large house. It consists of layers of detectors to pick up and identify the many energetic particles produced. All this to find out what went on in a region less than 10^{-15} m across, in which a huge energy was concentrated.

Many scattering experiments involve the electromagnetic interaction between charges. If the probe

• Quark models

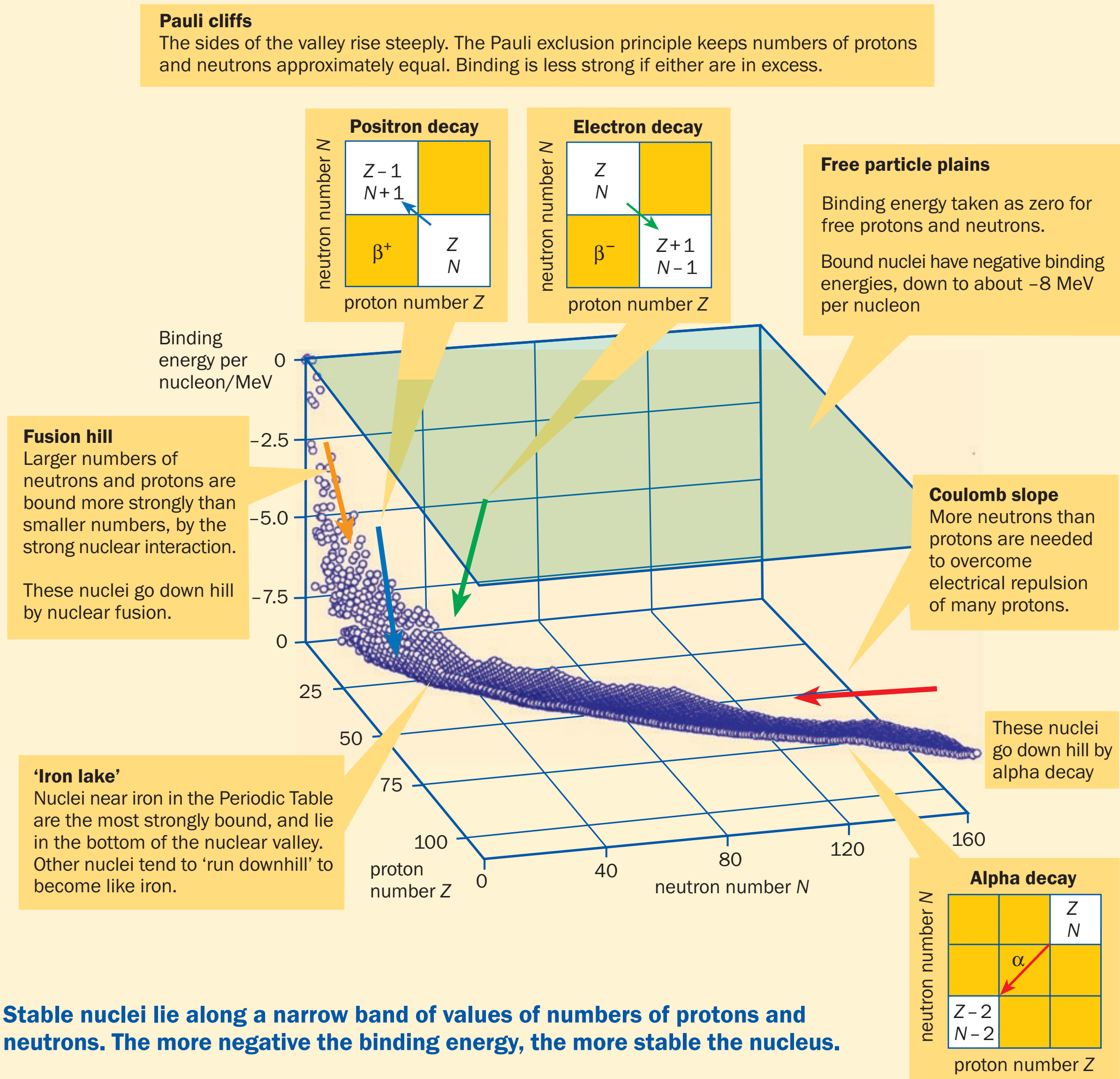
The valley of stability

A plot of the way binding energy varies with proton and neutron number looks like a deep narrow valley. Everywhere in the valley the binding energy is negative. The energy is zero up on the high flat plains around the valley, where the nucleus has been taken apart into free protons and neutrons.

The valley descends steeply to begin with, as extra neutrons and protons are added and the strong nuclear force takes effect. We have called this 'fusion hill' because it is the hill down which nuclei in stars go as heavy elements are made in nuclear fusion reactions (see page 224). The lowest point and strongest binding is around the element iron $^{56}_{26}$Fe, with binding energy –8.8 MeV per nucleon. This region we have called the 'iron lake', since lakes rest at the bottom of valleys. Beyond the iron lake, the valley floor rises gently, and the binding becomes weaker. This rise we have called the 'Coulomb slope', because the reduction in strength of binding is caused by the growing electrical Coulomb repulsion of the protons. Nuclei can fall down this slope too, back towards iron. Many do so by alpha decay, losing two protons and two neutrons in one go. Some suffer fission, breaking into smaller pieces (see page 221).

The valley has steep side walls, like a canyon. The graph shows the valley floor starting to turn up

Nuclear landscape: The nuclear valley of stability



Stable nuclei lie along a narrow band of values of numbers of protons and neutrons. The more negative the binding energy, the more stable the nucleus.

Questions

$c = 3 \times 10^8 \text{ m s}^{-1}$.

1 Describe the construction of any one form of fission reactor. In your account, be sure to:

(a) identify the fissile material used

(b) say whether the neutrons are moderated, and if so how

(c) say how the reactor is kept just critical

(d) explain how energy from the reaction is used to generate electrical power

(e) describe the necessary safety features of the design.

2 The rest energy associated with mass *m* is given by $E_{rest} = mc^2$. The table gives masses of some nuclei and the total mass of their neutrons and protons.

Nucleus	Mass of nucleus/ 10^{-27} kg	Mass of neutrons and protons/ 10^{-27} kg
$^{2}_{1}H$	3.343 64	3.347 60
$^{4}_{2}He$	6.644 77	6.695 21
$^{3}_{1}H$	5.007 44	5.022 56
$^{6}_{3}Li$	9.985 77	10.042 80

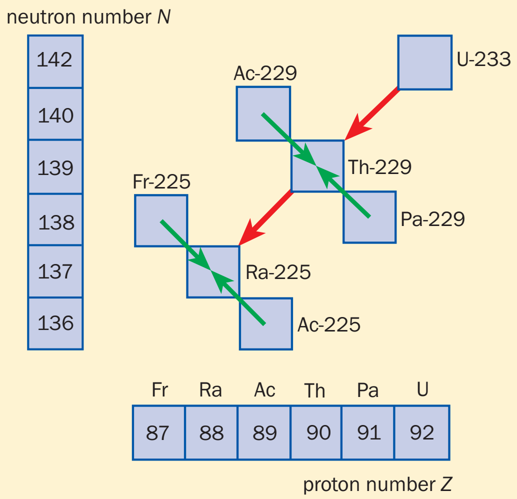
(a) Explain the significance of the fact that the values in the second column are larger than those in the first.

(b) Show that the rest energy of a mass of 1.0×10^{-27} kg is 562 MeV.

(c) Calculate the rest energy of the differences in mass between the first and second columns.

(d) Calculate the energy differences per nucleon, and arrange the four nuclei in order of stability.

3 Write equations for the six decays shown below. Indicate in each case the nature of the decay.



4 Draw some inferences about risks of ionising radiation from the following data, which show a typical European person's whole body dose equivalent per year from various sources.

Natural radiation	Dose equivalent/ μSv	% of total
Cosmic and solar radiation	310	13
Gamma radiation from soil, rocks, water	380	16
Radon decay products (in buildings)	800	33
Radiation from inside body (^{40}K, ^{14}C etc)	370	15
Total from natural sources	**1860**	**78**
Additional man-made radiation		
Medical and dental procedures	500	21
Weapons fall-out	10	0.4
Nuclear power	3	0.15
Occupational	9	0.36
Air travel	8	0.34
Total from man-made sources	**530**	**22**
Total from all sources	**2390**	**100**
Risk of developing a cancer approximately 5% per sievert		
Population of Britain approximately 50 million		

Radiation data from Dowsett D J, Kenny P A and Johnstone R E 1998 *The Physics of Diagnostic Imaging* (Chapman and Hall Medical).

5 Give your own example of a practical use of ionising radiation. In your account, be sure to:

(a) state how the ionising radiation is used in this case

(b) explain the particular choice of ionising radiation for this use

(c) describe the risks involved in its use and how they are dealt with

(d) indicate whether there are any alternatives to the use of ionising radiation in this case.

6 Suppose that the $^{236}_{92}U$ nucleus obtained when $^{235}_{92}U$ captures a neutron splits into two equal parts, 10^{-14} m apart.

(a) Show that the electrical potential energy of two protons 10^{-14} m apart is 0.14 MeV.

(b) Calculate the electrical potential energy of the two fragments.

(c) How might the existence of a strong nuclear attraction between the fragments explain why the answer to (b) is larger than the 200 MeV obtained from the fission of $^{235}_{92}U$?